

Exercise 42

Find the limit.

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\sin \theta}$$

Solution

Rewrite the limit in terms of ones that are known.

$$\begin{aligned} \lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\sin \theta} &= \lim_{\theta \rightarrow 0} \left(\frac{\cos \theta - 1}{\theta} \cdot \frac{\theta}{\sin \theta} \right) = \left(\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} \right) \left(\lim_{\theta \rightarrow 0} \frac{\theta}{\sin \theta} \right) \\ &= \left(\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} \right) \left(\lim_{\theta \rightarrow 0} \frac{1}{\frac{\sin \theta}{\theta}} \right) \\ &= \left(\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} \right) \left(\frac{1}{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}} \right) \\ &= (0) \left[\frac{1}{(1)} \right] \\ &= 0 \end{aligned}$$